Flight Testing of the Piper PA-28 Cherokee Archer II Aircraft

Emil Johansson   Fredrik Unell

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Abstract

It is sometimes easily assumed that an experimental measurement will closely mimic the results from an associated theoretical model. The purpose of this project is to determine how close to a theoretical model —involving the component buildup method and the quadratic drag polar assumption— a Piper Archer II will perform in an actual test flight.

During the test flight, the Archer II showed a similar correlation between airspeed and performance as the model. The actual performance numbers were however consistently lower than their theoretical counterpart.

Introduction

The primary question this report seeks to answer is if an experimental measurement of an aircraft's performance in the form of a test flight closely resembles the performance determined from a model consisting of a component buildup method and a quadratic drag polar assumption.

The aircraft that we will be examining and test flying —the PA-28-181— is a variant of the Piper PA-28 more commonly known as the Piper Cherokee Archer II. The Piper PA-28-181 will henceforth be referenced to as the *Archer II*.

The Piper PA-28 aircraft has since its introduction in 1960, by the Piper Aircraft manufacturer, been one of the most iconic and widely used general aviation aircraft. There is still to this day variants of the PA-28 in production and the total number of aircraft that has been built is greater than 32,000.

The Archer II variant is like the original PA-28 design a single engined, piston powered, low-wing aircraft with a conventional tricycle landing gear. The variant was certified in 1975 and is powered by a Lycoming O-360-A4A four-cylinder engine rated at 180 horsepower at 2700 rpm and features a laminar airfoil semi-tapered wing.

The performance of the Piper PA-28-181 aircraft will be experimentally analysed in terms of maximum climb rate, glide ratio and stall speed through a testflight in the Archer II conducted by ourselves. The result will be compared both to a theoretical model and the corresponding performance numbers given in the aircraft's *Pilot Operating Handbook*.

Contents

Nomenclature

Parameter	Description
C_D	Drag coefficient
C_{D0}	Zero-lift drag coefficient
K	Drag-duo-to-lift ciefficient
C_L	Lift coefficient
C_F	Component flat-plate skin friction coefficient
F_c	Component form factor
Q_c	component Interference factor
$S_{wet,c}$	Component wetted area
Re	Reynolds number
ρ	Density
V	Speed
l_c	Component length
μ	Viscosity
k	Skin roughness
f	Relative thickness
$(x/c)_m$	the chord-wise position of the maximum thickness
$\left(\frac{t}{c}\right)$	Maximum thickness normalized chord
Λ_m	Sweep of the wing
γ	Dihedral angle or climb angle
e_0	Oswald efficiency factor
AR	Aspect Ratio
R/C	Rate of Climb
W	Aircraft weight
n	Rotational speed [rpm]
D	Propeller diameter
P_{eng}	Engine power
η_{pr}	Propeller efficiency

Table 1: Nomenclature

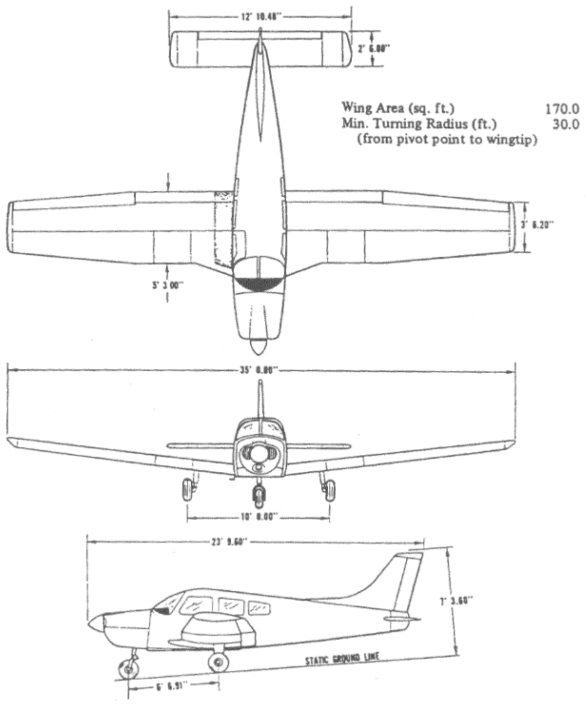

Figure 1: Piper Archer II

1 Performance Analysis

The first step when determining the performance of an aircraft is to calculate the drag coefficient as it is the base for all subsequent equations. The drag coefficient is based on the geometry of the aircraft, e.g. wing-areas and component lengths.

1.1 Estimate Drag coefficient

The drag coefficient, C_D, can be calculated with the use of the zero-lift drag coefficient, the drag-due-to-lift factor and the lift coefficient as follows.

$$C_D = C_{D0} + KC_L^2 \tag{1}$$

The zero-lift drag coefficient is calculated based on the geometry of the aircraft, the drag-due-to-lift coefficient and the lift coefficient is given from the geometry of the wing.

1.1.1 Zero-lift drag coefficient

There are two different methods to estimate the zero-lift drag coefficient, C_{D0}. The most simple method is *the equivalent skin-friction method* which is based on the total wetted area and a skin-friction coefficient for different aircraft classes. The other method is *the component buildup method* which is a little more detailed as it is based on parameters that is calculated for the specific aircraft. In this project *the component buildup method* have been chosen due to the fact that the geometry of the aircraft is known [5].

$$C_{D0} = \frac{1}{S}\sum_{c}[C_{F,c}F_cQ_cS_{\text{wet},c}] + \Delta C_{D,\text{misc}} + \Delta C_{D,L\&P} \tag{2}$$

The parameters $C_{F,c}$, F_c, Q_c and $S_{\text{wet},c}$ is calculated according to given formulas [1] where the geometry of the aircraft is taken into consideration. $\Delta C_{D,\text{misc}}$ is taken from a table of known features of the Archer II [4][p. 179].

1.1.2 The component flat-plate skin friction coefficient

The flat-plate skin friction coefficient, $C_{F,c}$, is based on the stream wise length of the component being studied and can be calculated according to equation (3).

$$C_F = \frac{0.455}{[log_{10}Re_{lc}]^{2.58}} \tag{3}$$

where Re_{lc} can be calculated from equation (4) or (5) depending on which one is the lowest.

$$Re_{lc} = \frac{\rho V l_c}{\mu} \tag{4}$$

$$Re_{cutoff} = 38.21\left(\frac{l_c}{k}\right)^{1.053} \tag{5}$$

where k is the skin roughness of the surface.

1.1.3 The component form factor

The component form factor, F_c, is calculated in different ways for different components. For components like the fuselage this factor is calculated according to equation (6)

$$F_c = 1 + \frac{60}{f^3} + \frac{f}{400} \tag{6}$$

where f is the relative thickness of the component

$$f = \frac{l_c}{d} \tag{7}$$

while wings and stabilisers are calculated according to equation (8)

$$F_c = \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34Ma^{0.18}(\cos\Lambda_m)^{0.28}\right] \tag{8}$$

where t/c is the maximum thickness normalized chord, $(x/c)_m$ is the chord-wise position of the maximum thickness and Λ_m is the sweep of this line.

1.1.4 Wetted area

The wetted area, $S_{wet,c}$, is the area which is in contact with the external airflow. This area is calculated differently for different components. Wings are calculated according to equation (10) and the fuselage is calculated according to equation (9).

$$S_{wet,c} = a\frac{1}{2}(A_{top} + A_{side}) \tag{9}$$

where a is a numeric constant, commonly used as 3.4 for common cross sections. A_{top} is the topside area of the fuselage and A_{side} is the side area of the fuselage.

$$S_{wet,c} = [1.977 + 0.52(t/c)]\,S_{exposed} \tag{10}$$

where $S_{exposed}$ is the exposed wing area. If the wing have a dihedral angle the exposed area is calculated according to (11).

$$S_{exposed} = \frac{S_{net}}{\cos\gamma} \tag{11}$$

1.1.5 Drag-due-to-lift factor

The drag-due-to-lift factor, K, is expressed as a function of the aspectratio, AR and Oswald efficiency factor, e_0.

$$K = \frac{1}{\pi A R e_0} \tag{12}$$

where e_0 is calculated from (13)

$$e_0 = 1.78(1 - 0.045AR^{0.68}) - 0.64 \tag{13}$$

1.1.6 Lift coefficient

To be able to calculate the lift coefficient for any given altitude and speed the following relation can be used.

$$C_L = \frac{mg}{\frac{1}{2}\rho V^2 S} \tag{14}$$

where ρ can be determined from the current altitude and temperature, V is the current speed and S is the area of the wing.

1.2 Glide ratio

The glide ratio can be determined from the relation between the lift coefficient and the drag coefficient.

$$glide\ ratio = \frac{C_L}{C_D} = \frac{C_L}{C_{D0} + KC_L} \tag{15}$$

With this relation where the lift coefficient is a changing parameter dependent on speed and altitude the glide performance can be determined for all possible scenarios.

1.3 Climb performance

Climb performance is about more than just the maximum rate of climb, the climb angle also needs to be taken into consideration as they give different results. The speed for best climb angle does not have to be the speed for best for rate of climb[1].

1.3.1 Rate of climb

The rate of climb parameter, R/C, is a parameter that depends on the current conditions. It is dependent on the current speed, air density and weight of the aircraft. It can be calculated according to [2] as

$$R/C = \frac{\eta_{pr}(V)P_{eng}(\rho)}{W} - C_D\frac{1}{2}\rho V^3\left(\frac{W}{S}\right)^{-1} - K\frac{2\left(\frac{W}{S}\right)cos^2\gamma}{\rho V} \tag{16}$$

where η_{pr} is the propeller efficiency and $P_{eng}(\rho)$ is the engine power at the current altitude and is calculated according to equation (17).

$$P_{eng}(\rho) = P_{eng,MSL}(1.13\sigma(\rho) - 0.13) \tag{17}$$

[1] They are in fact never the same speed except if the aircraft is flying at its absolute ceiling altitude

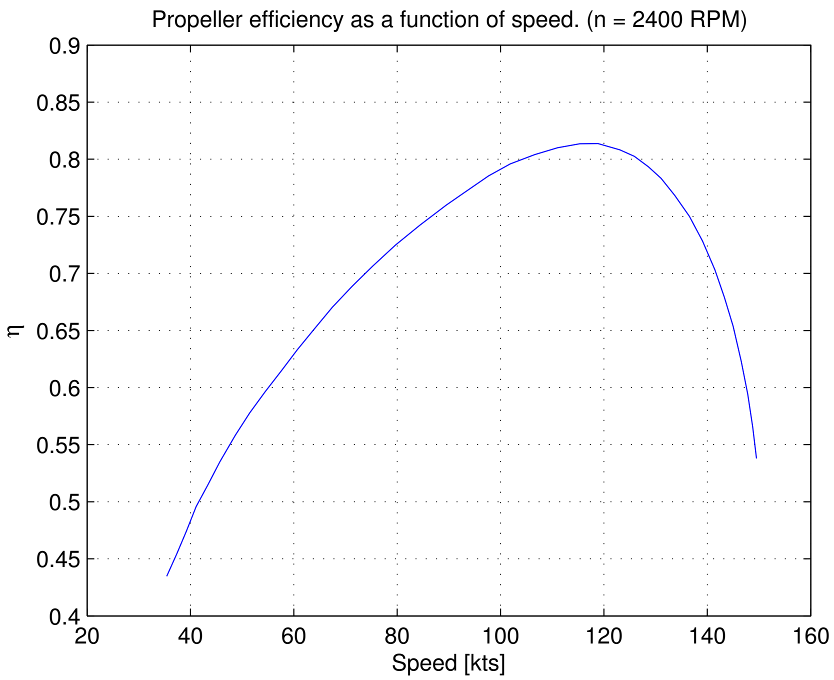

Figure 2: Propeller efficiency curve of the Archer II at 2400 rpm.

where σ is based on the relation between the density at the current altitude and the density at sea level as seen in equation (18).

$$\sigma(\rho) = \frac{\rho}{\rho_{MSL}} \tag{18}$$

The advance ratio, J, can be calculated from the airspeed, propeller diameter and rotational speed of the propeller according to equation (19).

$$J = \frac{V_\infty}{nD} \tag{19}$$

The relation between the propeller efficiency and advance ratio for the Archer II can be found in a table [4] where the propeller efficiency can be determined from a corresponding advance ratio. When assigning a rotational speed of 2400 rpm, the airspeed becomes the only unknown parameter. As the airspeed is the only unknown parameter the propeller efficiency can be plotted as a function of the airspeed as seen in figure 2.

1.3.2 Climb angle

The climb angle, γ, is the best angle of climb and can be calculated according to [2] as

$$\gamma = \arcsin\left(\frac{\eta_{pr}(V)P_{eng}(\rho)}{WV} - C_D\frac{1}{2}\rho V^2\left(\frac{W}{S}\right)^{-1} - K\frac{2\left(\frac{W}{S}\right)cos^2\gamma}{\rho V^2}\right) \tag{20}$$

1.4 Stall speed

The stall speed is the speed where the wings of the aircraft stall and looses its capacity to carry the weight of the aircraft. By rewriting equation (14) the stall speed can be determined as a function of CL.

$$V_s = \sqrt{\frac{2 \cdot L}{C_{L,\max} \cdot \rho \cdot S}} \tag{21}$$

2 Flight test

In addition to the theoretical analysis of the performance of the Archer II, a test flight has been performed. The purpose of the test flight was to examine and compare the data from the flight with the corresponding analytically calculated data.

The test flight included four elements; a maximum climb test, a glide test and a stall speed test.

2.1 The Atmosphere

Our planets atmosphere is varying in terms of air density at different locations, altitudes and at different times. An aircraft's performance is also affected by the air density in which it moves. In order to standardize and benchmark an aircraft's performance, the *International Standard Atmosphere* –henceforth referenced to as the *ISA*– is therefore commonly used. The aircraft's performance is generally presented as the performance it would have in this standard atmosphere.

Ideally, the aircraft's performance would be determined by test flying it in the ISA. The ISA is however a desk construction and it rarely –if ever– appears in the real world. Because of this, the aircraft's performance needs to be converted to the corresponding values in the ISA from the values gathered in the actual atmosphere at the time of the test flight.

The ISA specifies the standard pressure and temperature in the atmosphere. The altimeter inside the aircraft measures the static pressure on the outside and is calibrated in accordance with how the pressure varies with altitude in the ISA[2]. In usual cases, the altimeter setting is set to the actual pressure at sea level but for our test flight it is set to the standard setting if 1013 hPa (or 29.92 inHg). This means that the altimeter will show the *pressure altitude*, which is the altitude in which the ISA altitude has the same pressure as exists outside of the aircraft. Note that this is not the actual altitude of the aircraft, except if it flies in an atmosphere with identical pressure variation as the ISA.

Knowing the pressure altitude does not however mean that the density of the air is known because the density of the air does also depend on the temperature [3]. The compensate for any temperature deviations from the ISA and calculate the density altitude John T.

[2] `https://en.wikipedia.org/wiki/Altimeter#Use_in_aircraft`

[3] The density of the air does in fact also depend on humidity. The more moist the air is, the less dense it is since the molecular weight of water is lighter than that of air. This effect is however negliable in our climate.

Lowry suggests the following method, called the inchworm method [3].

To derive the method one starts with the atmospheric hydrostatic differential equation

$$\mathrm{d}p = -\rho g \mathrm{d}h \tag{22}$$

combined with the ideal gas law,

$$p = Rg\rho T \tag{23}$$

gives

$$\frac{-RT(p)\mathrm{d}p}{p} = \mathrm{d}h \tag{24}$$

The pressure altitude, h_p, can be defined as follows

$$h_p = \frac{T_0}{\alpha}\left(1 - \left(\frac{p}{p_0}\right)^{\alpha R}\right) \tag{25}$$

α —the temperature lapse rate— , p_0 —the pressure at sea level— and T_0 —the temperature at sea level— are all defined parameters in the ISA ($\alpha = 0.00650$ C/m, $p_0 = 1013.25$ hPa and $T_0 = 15\,^{\circ}\mathrm{C}$).

Differentiating (25) gives

$$\frac{\mathrm{d}h_p}{\mathrm{d}p} = -\frac{T_0 R}{p_0}\left(\frac{p}{p_0}\right)^{\alpha R - 1} \tag{26}$$

and combining (26) and (24) results in the following expression

$$\frac{T(p)}{T_0} = \left(\frac{p_0}{p}\right)^{\alpha R} \mathrm{d}h_p = \mathrm{d}h \tag{27}$$

.

Using

$$T_S(h) \equiv T_0 - ah, \tag{28}$$

where h is height in the ISA, it follows from the ideal gas law (23) that

$$\frac{\mathrm{d}p}{p} = -\frac{\mathrm{d}h}{R(T_0 - \alpha h)}$$

and by integration

$$p_S(h) = p_0\left(1 - \frac{\alpha h}{T_0}\right)^{1/\alpha R} \tag{29}$$

By division, (28) and (29) gives the relationship between sea level and another position in the standard atmosphere as

$$\frac{T_0}{T_S(h_p)} = \left(\frac{p_0}{p}\right)^{\alpha R} \tag{30}$$

Combining (30) and (26) results in

$$\mathrm{d}h = \frac{T(p)}{T_S(h_p)}\mathrm{d}h_p$$

and after integration

$$\Delta h = \int_{h_{p1}}^{h_{p2}} \frac{T(p)}{T_S(h_p)}\mathrm{d}h_p \tag{31}$$

.

With (31) it is possible to calculate the height difference in the standard atmosphere, Δh, given the temperature at the various pressure altittudes, $T(p)$ and the corresponding temperatures in ISA, $T(h_p)$. It is however considerably easier and more practical to evaluate the integral in (31) by using the mean values at the ends of the interval:

$$\Delta h = \left[\frac{T(h_{p1})}{T_S(h_{p1}} + \frac{T(h_{p2})}{T_S(h_{p2}}\right] \cdot \frac{\Delta h_p}{2}. \tag{32}$$

(32) is a good approximation since the temperature generally varies slowly and steadily at the pressure altitude ranges we will be concerned with.

In practice, this means that only the temperature at the beginning of the pressure altitude range, $T(h_{p1})$, and at the end of the range, $T(h_{p2})$, would have to be recorded during the test flight, together with the pressure altitudes themselves.

2.2 Testing Procedure

At the day and time of the test flight, ground temperature and gross weight of the airplane were noted and the altimeter was set to 1013 hPa as to read pressure altitude directly. Once the pre-flight checks and procedures were completed and the aircraft was airborne the testing could begin.

The maximum climb test was conducted as follows; the aircraft's throttle was kept fully open and the aircraft's indicated airspeed was kept constant during the steady vertical climb. The time to climb between two altitudes and the temperature at those altitudes were noted as well as the propellers rpm. The glide test was conducted in a similar manner to the maximum climb test with the exception that the throttle was kept closed while the aircraft descended at idle power through the two altitudes.

The maximum climb and glide procedures were repeated several times at various airspeeds, increasing with 10 kt increments.

The stall speed was noted in a idle power, level stall.

The flight instruments , such as the speed-indicator and altimeter, were recorded with a video camera during the entire flight for further analysis of the data on the ground.

2.3 Flight Data Analysis

Once on the ground again, the flight data were analysed. Δh was calculated in accordance with (32) and divided with the recorded climb time to obtain the equivalen rate of climb in the standard atmosphere

$$\text{Rate of Climb} = \frac{\Delta h}{t}.$$

The climb angles were obained by

$$\text{Climb Angle} = \sin^{-1}\left(\frac{\text{Rate of Climb}}{\text{Indicated Airspeed}}\right).$$

The sink rates in the glide tests were calculated in a similar manner to the rates of climb in the maximum climb tests. Additionally, to obatain the glide ratios, the following calculation was made

$$\text{Glide Ratio} = \frac{C_L}{C_D} = \frac{\text{Indicated Airspeed}}{\text{Sink rate}}$$

In order to compare our testflight values with the theoretically calculated ones, new theoretical calculations were made. This time with parameter values for *air density* and *aircraft gross weight* matching those of our flight test conditions.

3 Results

3.1 Theoretical performance

The equations 3-14 give the results for all components in table 2.

	Fuselage	Main wing	Vertical tail	Horizontal tail
F_C	1.2544	1.2762	1.2100	1.2607
C_F	0.0026	0.0033	0.0031	0.0038
Q	1.000	1.2000	1.0500	1.0500
S_{wet}	18.1607	32.4690	2.1710	5.0169

Table 2: Component values

Using these values into equation (2) results in the final sum for $C_{D0} = 0.0296$.

From the theoretical analysis of the aircraft's performance the following data were computed: The glide ratio as a function of the aircraft's indicated airspeed is shown in figure 3. The maximum climb rate at various airspeeds is represented in figure 4 and figure 5 shows how the climb angle is varying with airspeed. The airspeed for best rate of climb, V_y, was calculated as 78.6 knots. The Pilot Operating Handbook does under the same conditions present a value of 76 knots [6]. The Archer II will, according to the POH, climb at a maximum rate of 735 feet/min at sea level and maximum gross weight. The comparable value from the theoretical performance model is 1040 feet/min.

V_x, the airspeed for best climb angle is lower, 64 knots according to the POH and 55 knots according to our theoretical model.

The best glide ratio for the Archer II is, with the theoretical model used 12.6, higher than the value provided by the POH, which is 10.1.

The stall speed of the Archer II is 53 knots at a flaps-up configuration at sea level and MTOW according to the POH. Our calculations give a stall speed at the same circumstances of 59 knots.

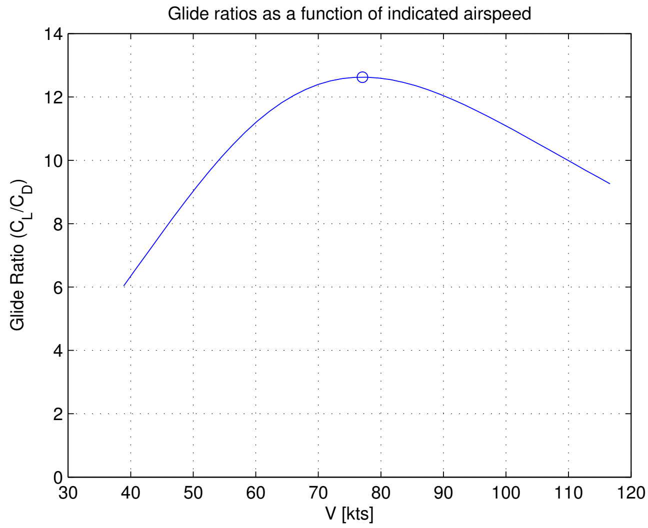

Figure 3: The glide ratio as a function of indicated airspeed at sea level in ISA with MTOW (1156 kg).

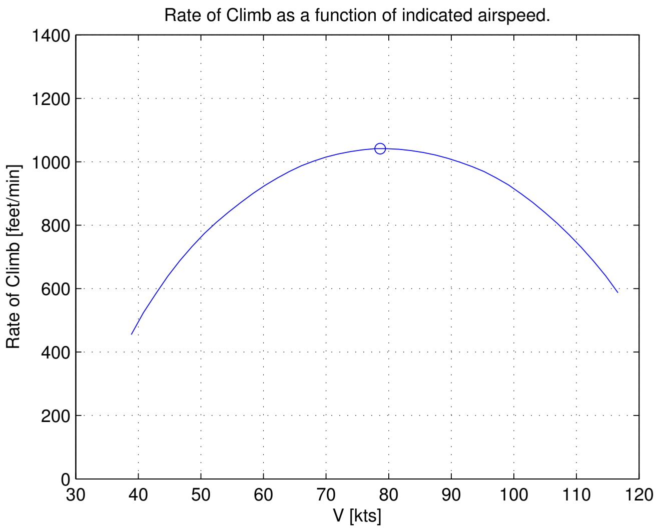

Figure 4: Rate of Climb as a function of indicated airspeed at sea level in ISA with MTOW.

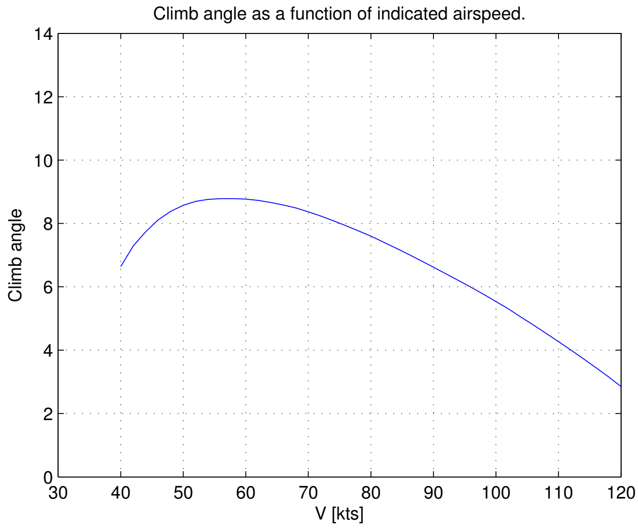

Figure 5: Climb angle as a function of indicated airspeed at sea level in ISA with MTOW.

3.2 Test flight performance

The maximum climb and glide ratio tests during our test flight were all conducted between the pressure altitudes of 3000 and 4000 feet. The climb rates and glide ratios are therefore compared to the theoretical values with the average air density between those altitudes (corresponding to a density altitude of 3565 feet in this case). The actual weight of the aircraft at the time of the test flight was determined to be 924 kg so the theoretical values in the comparisons were calculated using that weight as well. The comparisons are presented in figure 6 for glide ratios, in figure 7 for climb rates and in figure 8 for climb angles.

The Archer II has a relatively unpronounced making it somewhat difficult to determine the exact stall speed during the test but the aircraft stalled at around 50 kts.

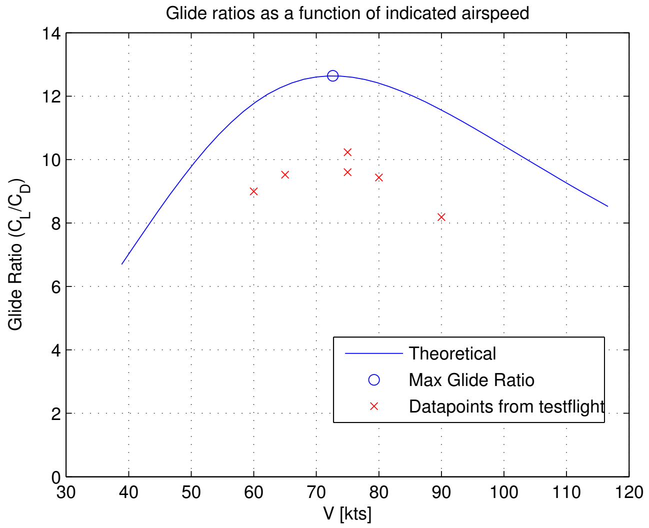

Figure 6: Glide ratios as a function of indicated airspeed combined with datapoints from the test flight at 3500 feet in ISA and with an aircraft weight of 924 kg.

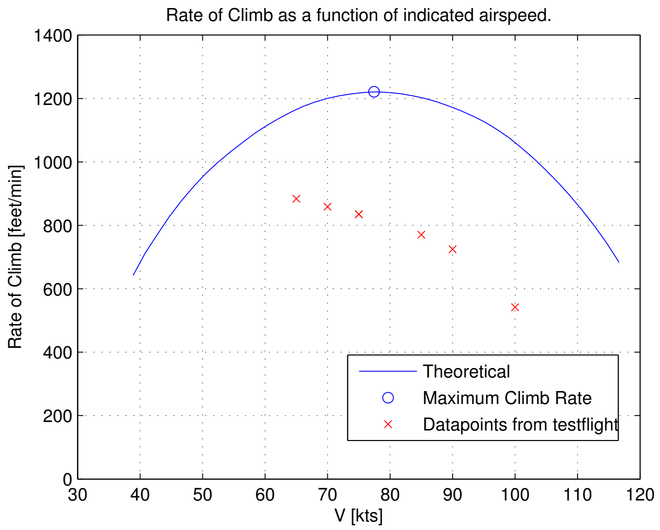

Figure 7: Rate of Climb as a function of indicated airspeed combined with datapoints from the test flight at 3500 feet in ISA and with an aircraft weight of 924 kg.

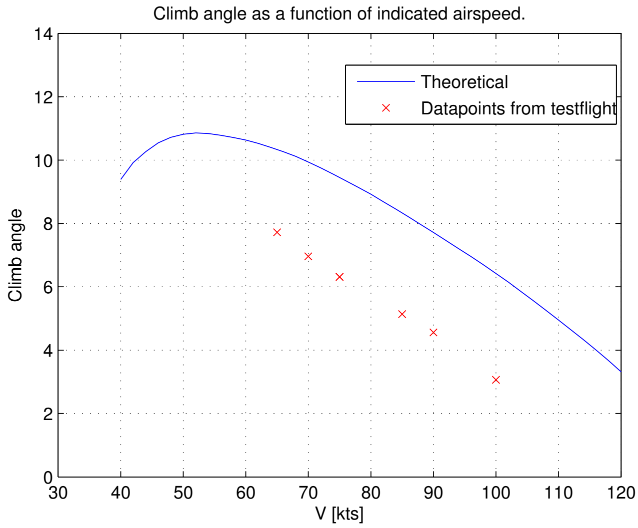

Figure 8: Climb angle as a function of indicated airspeed combined with datapoints from the test flight at 3500 feet in ISA and with an aircraft weight of 924 kg.

4 Discussion

The first test flight that was conducted took place in an erratic and turbulent atmosphere which resultet in unconsistent and unreliable data. It was therefore decided that another test flight should be performed. The second test flight occured under much more favorable conditions with more consistent measurements as a result. The measurements from the second flight are the only ones that are included in this report.

The distribution of the value from the glide ratio tests does resemble the theoretically calculated curve under the same conditions. The actual glide ratios are however all lower than their theoretical counterparts as seen in figure (6). The same tendency can be seen for the climb angles in figure 8 as well. The distribution is similar but the test flight performance numbers are lower.

The systematic tendency of the practical values to be lower could be explained with factors making the aircraft perform worse such as a dirty fuselage, less than ideal maneuvering from the pilot and in the case of the climb test; an aged and underperforming engine and/or propeller.

Other factors might include uncalibrated altimeter, thermometer, stop watch and/or airspeed indicator as well as erratic pressure and temperature in our altitude range. These factor might as well though induce errors with the opposite tendency.

The comparable values from the theoretical model at MTOW and the POH suggests that the theoretical model used gives an overestimation of the Archer II's performance. That would imply a smaller difference between the aircraft's optimal performance and the measured performance from our test flight than is shown in figure (6)-(8).

There is however, one scenario where the measured data does not match the pattern from the theoretical model; the maximum rate of climb scenario. The measured data implies a relationship were the rate of climb increases with decreased airspeed unlike the theoretical curve which has a conversely quadratic shape. This might be due to the effect the propeller slipstream has on the other parts of the aircraft.

When the propeller is turning at a high rate, a considerably air flow will be prodiced which blows past the inner section of the main wing. This effectively lowers the angle of attack of the inner part of the wing. This effect might give the impression of the aircraft performing better at lower airspeeds than expected, because the propeller slip stream in fact adds air flow over a big section of the wing making it perform as if the actual airspeed was higher. That would also explain why the aircraft seems to perform best at a higher airspeed during the glide test, in which the propeller slip stream effect is much smaller.

5 Conclusions

- The component buildup method gives an overestimation of the Archer II's performance compared to the Pilot's Operating Handbook
- The data measured from the test flight have a similar distribution as the values derived from the theoretical model.
- The measured data does consistently indicate lower performance than the theoretical model under the same circumstances
- Favourable conditions during the test flight is beneficial to the gathered measurements.

References

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Division of Labour

Emil has been responsible for the planning and conducting of flight testing and the part attributed to that in the report. He has also written the discussion and introduction parts in the report.

Fredrik has been responsible for the theoretical model and calculations and has written that part in the report as well.

With that said, most of the project has been a shared commitment and we have helped each other on all parts of the project.

Flight Test Data

Description	Max Climb	Max Climb	Max Climb	Max Climb	Max Climb	Max Climb	Max Clim	Max Climb	Glide	Glide	Glide	Glide	Glide	Glide
Speed	85	90	65	70	75	100	75	80	75	80	90	65	60	75
Start Level	3000	3000	3000	3000	3000	3000	3000	2500	3000	3000	3000	3000	3000	3000
End Level	4000	4000	4000	4000	4000	4000	4000	3000	2000	2000	2000	2000	2000	2000
Start Temp	50	50	50	50	50	50	50	53	50	50	50	50	50	50
End Temp	45	45	45	45	45	45	45	50	45	45	45	45	45	45
Time	01:18	01:23	01:08	01:10	01:12	01:51	01:12	00:34	01:16	01:10	00:54	01:27	01:29	1:21